

Vibration Analysis Summary of Common Formulae

English Language
US Customary and SI Units
Rev: 2026-05-29

Introduction:

The following pages contain a collection of equations, conversions, and other vibration related information. This information has been assembled primarily to aid examinees during the Vibration Institute's ISO 18436-2 based certification examinations and may also have value as a reference.

This "Summary of Common Formulae" is intended to be a resource for vibration analysts and will be provided in full for all online Vibration Institute certification examinations and paper Category IV examinations. (For Category I, II and III paper certification examinations, reduced equation sheets appropriate for the category exam are included. These reduced equation sheets are available for download at <https://www.vi-institute.org/vibration-analyst-equations/>).

*Personal copies of the "Summary of Common Formulae" shall **NOT** be used during a VI VA certification examination.*

The following pages include:

- A: Units*, Constants* and Conversions*
- B: Forces
- C: Motions
- D: Frequencies
- E: Signal Processing
- F: Vibration Response
- G: Stability Threshold
- H: Amplification Factor and Damping Ratio
- I: Material Properties*
- J: Area Moments of Inertia
- K: Mass Moments of Inertia
- L: Cases of Stiffness Calculation

Note:

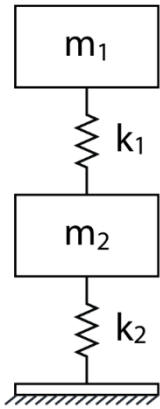
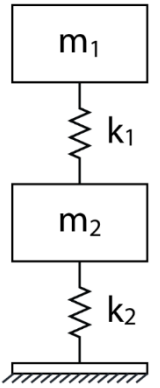
Numbers in this "Summary of Common Formulae" and in all Vibration Institute certification examinations typically are shown without commas and use a period character as the decimal marker to indicate where digits smaller than one (unity) begin. Example: 10023.7 is ten thousand twenty-three and seven tenths (no commas, one period).

	US Customary Units	SI Units
A	UNITS*, CONSTANTS* AND CONVERSIONS*	
① ② ③ ④	length: foot, ft; inch, in mass: slug, lbf s²/ft time: second, s force: pound force, lbf pressure: psi, lbf/in² acceleration: g, ft/s² or in/s² velocity: ft/s or in/s rotational speed: RPM, RPS, Hz, rad/s plane angle: degree, deg, ° full circle = 360° = 2π rad weight, W: lbf pi, π = 3.142 standard acceleration due to gravity, g_n = 32.17 ft/s² = 386.1 in/s² mass** = W/g_n 1 lbf = 0.2248 N 1 foot (ft) = 12 inches (in) 1 inch (in) = 1000 mils 1 inch (in) = 0.02540 meter = 25.4 mm 1 in² = 6.452 cm² 1 in³ = 16.39 cm³ 1 pound (lbf) = 16 ounces (oz) 1 ounce (oz)** = 28.35 grams (g) 1 psi = 6.895 kPa 1 lbf/in = 175.1 N/m = 0.1751 N/mm *conversions and constants shown may be approximate, rounded, or stated for the field of vibration. **where acceleration due to gravity = g_n	length: meter, m; millimeter, mm mass: kilogram, kg time: second, s force: Newton, N pressure, Pascal, 1 Pa = 1 N/m² acceleration: g, m/s² velocity: m/s rotational speed: RPM, RPS, Hz, rad/s plane angle: radian, rad full circle = 2π rad = 360° weight, W: N pi, π = 3.142 standard acceleration due to gravity, g_n = 9.807 m/s² 1 N = 4.448 lbf 1 m = 39.37 in = 1000 mm 1 mm = 0.03937 in 1 micron (μm) = 0.00003937 in = 0.0010 mm 1 cm² = 0.1550 in² 1 cm³ = 0.0610 in³ 1 kg = 2.205 lbf** 1 gram (g) = 0.0353 oz** 1 kPa = 0.1450 psi = 0.001 MPa 1 kN/m = 5.710 lbf/in = 1 N/mm *conversions and constants shown may be approximate, rounded, or stated for the field of vibration. **where acceleration due to gravity = g_n

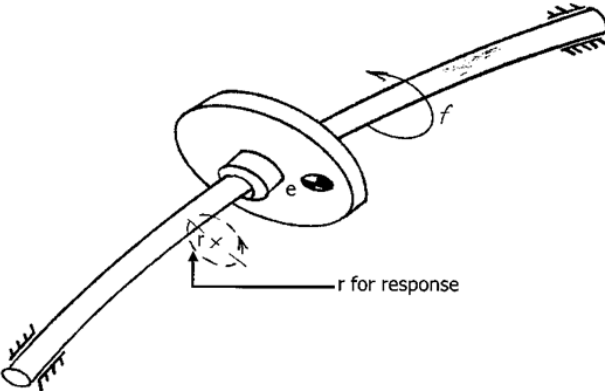
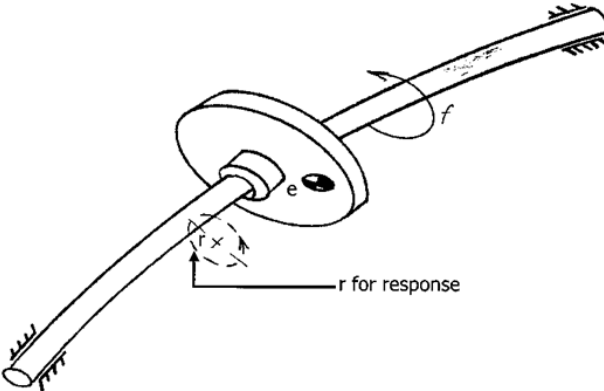
B FORCES		
2 3 4	Mass Unbalance Force (lbf) $F = Me \left(\frac{2\pi N}{60} \right)^2$ M = W/g _n , mass, lbf s ² /in W = weight of rotor or balance weight, lbf g _n = 386.1 in/s ² e = rotor eccentricity or radius of balance weight, in N = rotational speed, rpm	Mass Unbalance Force (Newtons) $F = Me \left(\frac{2\pi N}{60} \right)^2$ M = mass, kg e = rotor eccentricity or radius of balance mass, m N = rotational speed, rpm
2 3 4	Spring Force (lbf) $F = Kx$ K = stiffness of spring, lbf/in x = relative deflection, in	Spring Force (Newtons) $F = Kx$ K = stiffness of spring, N/m x = relative deflection, m
3 4	Damping Force (lbf) $F = C\dot{x}$ C = damping constant, lbf s/in $\dot{x}$ = relative velocity, in/s	Damping Force (Newtons) $F = C\dot{x}$ C = damping constant, N s/m $\dot{x}$ = relative velocity, m/s
3 4	Inertia Force (lbf) $F = M\ddot{x}$ M = mass, lbf s ² /in $\ddot{x}$ = acceleration, in/s ²	Inertia Force (Newtons) $F = M\ddot{x}$ M = mass, kg $\ddot{x}$ = acceleration, m/s ²
C MOTIONS		
1 2 3 4	Acceleration $A = V(2\pi f)$ A = acceleration, in/s ² V = velocity, in/s f = frequency in Hz (CPS)	Acceleration $A = \frac{V(2\pi f)}{1000}$ A = acceleration, m/s ² V = velocity, mm/s f = frequency in Hz (CPS)

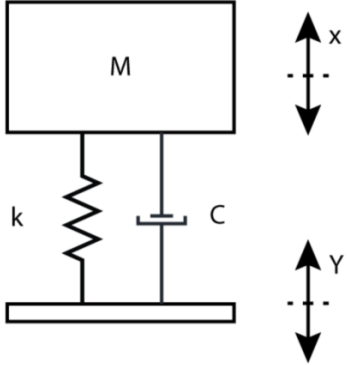
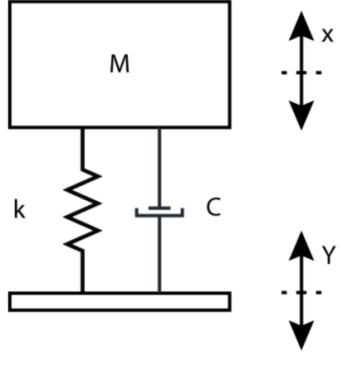
① ② ③ ④	Velocity (in/s) $V = D(2\pi f)$ D = peak displacement, in f = frequency, cycle/s (cps or Hz)	Velocity (mm/s) $V = D(2\pi f)$ D = peak displacement, mm f = frequency, cycle/s (cps or Hz)
① ② ③ ④	Displacement (in peak) $D = \frac{V}{2\pi f}$ $D = \frac{A}{(2\pi f)^2}$ V = velocity, in/s f = frequency in Hz (CPS) A = acceleration, in/s ²	Displacement (mm peak) $D = \frac{V}{2\pi f}$ $D = \frac{1000 A}{(2\pi f)^2}$ V = velocity, mm/s f = frequency in Hz (CPS) A = acceleration, m/s ²
D	FREQUENCIES	
② ③ ④	Natural Frequency – Classical $f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ k = stiffness, lbf/in m = W/g _n W = weight, lbf g _n = 386.1 in/s ² f _n = natural frequency of a single-degree-of-freedom system, Hz	Natural Frequency - Classical $f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ k = stiffness, N/m m = mass, kg f _n = natural frequency of a single-degree-of-freedom system, Hz
② ③ ④	Frequency Unit Conversion $\omega = 2\pi f$ ω = frequency, rad/s f = frequency, Hz or CPS	Frequency Unit Conversion $\omega = 2\pi f$ ω = frequency, rad/s f = frequency, Hz or CPS
② ③ ④	Roll Rotational Speed $\Omega = \frac{V}{5\pi D}$ V = web velocity, ft/min D = roll diameter, in Ω = rotational speed, RPS	Roll Rotational Speed $\Omega = \frac{16.66 V}{\pi D}$ V = web velocity, m/min D = roll diameter, mm Ω = rotational speed, Hz

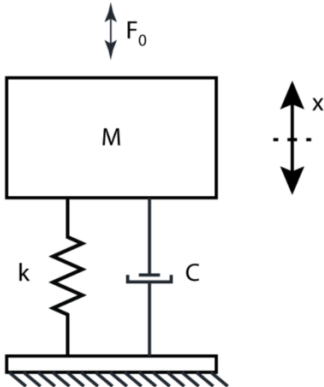
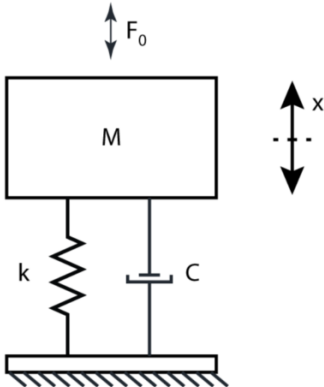
3 4	Natural Frequency - Static Deflection* $f_n = \frac{1}{2\pi} \sqrt{\frac{g_n}{\delta_{st}}}$ $g_n = 386.1 \text{ in/s}^2$ δ_{st} = static deflection, in f_n = natural frequency of single-degree-of-freedom system, Hz *Only valid for linear spring rate and where the static deflection is in-line with gravity.	Natural Frequency - Static Deflection* $f_n = \frac{1}{2\pi} \sqrt{\frac{g_n}{\delta_{st}}}$ $g_n = 9.807 \text{ m/s}^2$ δ_{st} = static deflection, m f_n = natural frequency of single-degree-of-freedom system, Hz *Only valid for linear spring rate and where the static deflection is in-line with gravity.
2 3 4	Bearing Frequencies $FTF_i = \left(\frac{\Omega}{2}\right) \left[1 - \left(\frac{B}{P}\right) \cos(CA)\right]$ $FTF_o = \left(\frac{\Omega}{2}\right) \left[1 + \left(\frac{B}{P}\right) \cos(CA)\right]$ $BPFI = \left(\frac{N}{2}\right) \Omega \left[1 + \left(\frac{B}{P}\right) \cos(CA)\right]$ $BPFO = \left(\frac{N}{2}\right) \Omega \left[1 - \left(\frac{B}{P}\right) \cos(CA)\right]$ $BSF = \left(\frac{P}{2B}\right) \Omega \left[1 - \left(\frac{B}{P}\right)^2 \cos^2(CA)\right]$ FTF = fundamental train frequency FTF_i = inner race rotating FTF_o = outer race rotating BPFI = ball pass frequency, inner race BPFO = ball pass frequency, outer race BSF = ball spin frequency CA = contact angle, deg Ω = rotating race speed (Hz or RPM) N = number of rolling elements per row P = pitch diameter, in B = ball or roller diameter, in	Bearing Frequencies $FTF_i = \left(\frac{\Omega}{2}\right) \left[1 - \left(\frac{B}{P}\right) \cos(CA)\right]$ $FTF_o = \left(\frac{\Omega}{2}\right) \left[1 + \left(\frac{B}{P}\right) \cos(CA)\right]$ $BPFI = \left(\frac{N}{2}\right) \Omega \left[1 + \left(\frac{B}{P}\right) \cos(CA)\right]$ $BPFO = \left(\frac{N}{2}\right) \Omega \left[1 - \left(\frac{B}{P}\right) \cos(CA)\right]$ $BSF = \left(\frac{P}{2B}\right) \Omega \left[1 - \left(\frac{B}{P}\right)^2 \cos^2(CA)\right]$ FTF = fundamental train frequency FTF_i = inner race rotating FTF_o = outer race rotating BPFI = ball pass frequency, inner race BPFO = ball pass frequency, outer race BSF = ball spin frequency CA = contact angle, deg Ω = rotating race speed (Hz or RPM) N = number of rolling elements per row P = pitch diameter, mm B = ball or roller diameter, mm

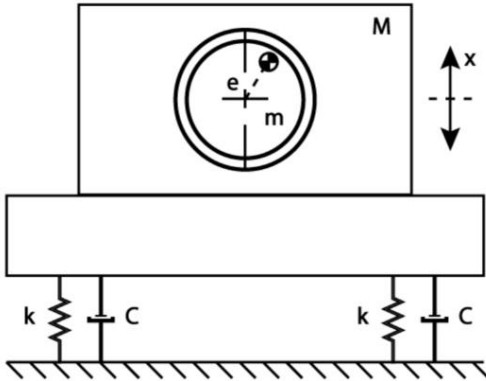
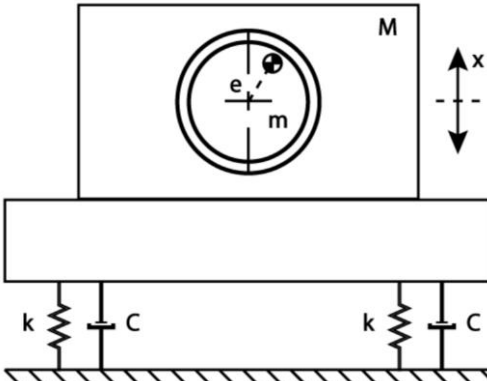
1 2 3 4	General Guidelines For Bearing Frequencies* BPFO = 0.41 x N x Ω BPFI = 0.59 x N x Ω FTF_i = 0.41 x Ω** FTF_o = 0.59 x Ω** BSF = 0.22 x N x Ω N = number of rolling elements per row Ω = speed units of RPM or Hz *for use in F_{Max} selection ONLY. ** FTF_i applies when inner race rotates; FTF_o applies when outer race rotates	General Guidelines For Bearing Frequencies* BPFO = 0.41 x N x Ω BPFI = 0.59 x N x Ω FTF_i = 0.41 x Ω FTF_o = 0.59 x Ω BSF = 0.22 x N x Ω N = number of rolling elements per row Ω = speed units of RPM or Hz *for use in F_{Max} selection ONLY. ** FTF_i applies when inner race rotates; FTF_o applies when outer race rotates
4	Natural Frequencies – Undamped 2 Degree of Freedom $\omega^2 = \frac{k_2 + k_1}{2m_2} + \frac{k_1}{2m_1} \pm \sqrt{\frac{1}{4} \left[\frac{k_2 + k_1}{m_2} + \frac{k_1}{m_1} \right]^2 - \frac{k_2 k_1}{m_2 m_1}}$ $f_{1,2} = \frac{1}{2\pi} \omega_{1,2}$ k₁, k₂ = stiffness, lbf/in m₁, m₂ = mass, lbf-s²/in ω = natural frequency, rad/s 	Natural Frequencies – Undamped 2 Degree of Freedom $\omega^2 = \frac{k_2 + k_1}{2m_2} + \frac{k_1}{2m_1} \pm \sqrt{\frac{1}{4} \left[\frac{k_2 + k_1}{m_2} + \frac{k_1}{m_1} \right]^2 - \frac{k_2 k_1}{m_2 m_1}}$ $f_{1,2} = \frac{1}{2\pi} \omega_{1,2}$ k₁, k₂ = stiffness, N/m m₁, m₂ = mass, kg ω = natural frequency, rad/s 

E	SIGNAL PROCESSING	
① ② ③ ④	Peak vs. RMS* peak = 1.414 rms *Applies to single frequency sine waveforms only.	Peak vs. RMS* peak = 1.414 rms *Applies to single frequency sine waveforms only.
① ② ③ ④	Default Frequency Spans Operating Speed $F_{MAX} \geq 10 \times \text{RPM}$ Rolling Element Bearings $F_{MAX} \geq 10 \times \text{BPFI}$ Fluid Film Bearings $F_{MAX} \geq 10 \times \text{RPM}$ Vane/Blade Pass $F_{MAX} \geq 3 \times \# \text{ Vanes/Blades} \times \text{RPM}$ Electrical $F_{MAX} \geq 3 \times 2X \text{ Line Frequency}$ Gear Mesh $F_{MAX} \geq 3 \times \text{Gear Mesh Frequency}$ RPM = rotational speed F_{MAX} = maximum frequency	Default Frequency Spans Operating Speed $F_{MAX} \geq 10 \times \text{RPM}$ Rolling Element Bearings $F_{MAX} \geq 10 \times \text{BPFI}$ Fluid Film Bearings $F_{MAX} \geq 10 \times \text{RPM}$ Vane/Blade Pass $F_{MAX} \geq 3 \times \# \text{ Vanes/Blades} \times \text{RPM}$ Electrical $F_{MAX} \geq 3 \times 2X \text{ Line Frequency}$ Gear Mesh $F_{MAX} \geq 3 \times \text{Gear Mesh Frequency}$ RPM = rotational speed F_{MAX} = maximum frequency
② ③ ④	Frequency Resolution* Frequency resolution = (frequency span x window noise factor x 2) / # of FFT lines Window Noise Factor: 1.0 for uniform, rectangular, “no window” 1.5 for Hanning window 3.8 for flat top window *ability to resolve closely spaced signals	Frequency Resolution* Frequency resolution = (frequency span x window noise factor x 2) / (# of FFT lines) Window Noise Factor: 1.0 for uniform, rectangular, “no window” 1.5 for Hanning window 3.8 for flat top window *ability to resolve closely spaced signals
② ③ ④	Block Data Acquisition Time (DAT)* DAT = # FFT lines/frequency span *Applicable when Nyquist Factor = 2.56	Block Data Acquisition Time (DAT)* DAT = # FFT lines/frequency span *Applicable when Nyquist Factor = 2.56

③ ④	Dynamic Range $DR = 20 \log \frac{V_m}{V_r}$ $\frac{V_m}{V_r} = 10^{\frac{dB}{20}}$ V_m = voltage measured V_r = voltage reference DR = dynamic range, dB (decibels)	Dynamic Range $DR = 20 \log \frac{V_m}{V_r}$ $\frac{V_m}{V_r} = 10^{\frac{dB}{20}}$ V_m = voltage measured V_r = voltage reference DR = dynamic range, dB (decibels)
F VIBRATION RESPONSE		
④	Rotor Vibration Response  $r = \frac{e \left(\frac{f}{f_n} \right)^2}{\sqrt{\left[1 - \left(\frac{f}{f_n} \right)^2 \right]^2 + \left[2\zeta \frac{f}{f_n} \right]^2}}$ r = rotor vibration response, in peak e = eccentricity of mass, in f = forcing frequency, Hz f_n = natural frequency, Hz ζ = damping ratio, C/C_C C = damping constant, lbf s/in C_C = critical damping = $2 m \omega_n$ m = mass, lbf s²/in $\omega_n = \sqrt{\frac{k}{m}}$, rad/s	Rotor Vibration Response  $r = \frac{e \left(\frac{f}{f_n} \right)^2}{\sqrt{\left[1 - \left(\frac{f}{f_n} \right)^2 \right]^2 + \left[2\zeta \frac{f}{f_n} \right]^2}}$ r = rotor vibration response, mm peak e = eccentricity of mass, mm f = forcing frequency, Hz f_n = natural frequency, Hz ζ = damping ratio, C/C_C C = damping constant, N s/m C_C = critical damping = $2 m \omega_n$ m = mass, N s²/m (= kg) $\omega_n = \sqrt{\frac{k}{m}}$, rad/s

4	Phase $\tan \phi = \frac{2\zeta \frac{f}{f_n}}{1 - \left(\frac{f}{f_n}\right)^2}$ ϕ = phase, deg f = forcing frequency, Hz f_n = natural frequency, Hz	Phase $\tan \phi = \frac{2\zeta \frac{f}{f_n}}{1 - \left(\frac{f}{f_n}\right)^2}$ ϕ = phase, deg f = forcing frequency, Hz f_n = natural frequency, Hz
4	Transmissibility  $\frac{X}{Y} = \frac{F_{TR}}{F_0} = \frac{\sqrt{1 + \left[2\zeta \frac{f}{f_n}\right]^2}}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$ F_{TR} = transmitted force, lbf F_0 = exciting force, lbf f = forcing frequency, Hz f_n = natural frequency, Hz ω_n = natural frequency, rad/s X = mass motion, in peak Y = base motion, in peak ζ = damping ratio = C/C_c k = spring stiffness, lbf/in C = damping constant, lbf s/in $C_c = 2m\omega_n$ = critical damping, lbf s/in M = mass, lbf-s²/in	Transmissibility  $\frac{X}{Y} = \frac{F_{TR}}{F_0} = \frac{\sqrt{1 + \left[2\zeta \frac{f}{f_n}\right]^2}}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$ F_{TR} = transmitted force, N F_0 = exciting force, N f = forcing frequency, Hz f_n = natural frequency, Hz ω_n = natural frequency, rad/s X = mass motion, mm peak Y = base motion, mm peak ζ = damping ratio = C/C_c k = spring stiffness, N/m C = damping constant, N s/m $C_c = 2m\omega_n$ = critical damping, N s/m M = mass, kg

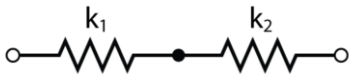
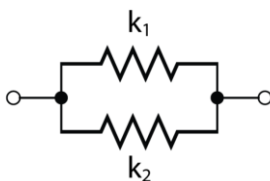
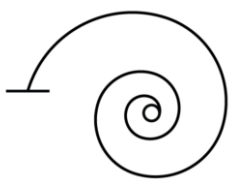
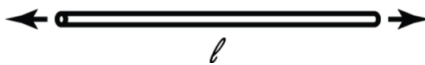
4	Forced Vibration Response  $X = \frac{\frac{F_0}{k}}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$ F_0 = exciting force, lbf f = forcing frequency, Hz f_n = natural frequency, Hz ω_n = natural frequency, rad/s X = mass motion, inches peak ζ = damping ratio = C/C_c k = spring stiffness, lbf/in C = damping constant, lbf s/in $C_c = 2M\omega_n$ = critical damping, lbf s/in M = mass, lbf s²/in	Forced Vibration Response  $X = \frac{\frac{F_0}{k}}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$ F_0 = exciting force, N f = forcing frequency, Hz f_n = natural frequency, Hz ω_n = natural frequency, rad/s X = mass motion, mm peak ζ = damping ratio = C/C_c k = spring stiffness, N/m C = damping constant, N s/m $C_c = 2M\omega_n$ = critical damping, N s/m M = mass, kg
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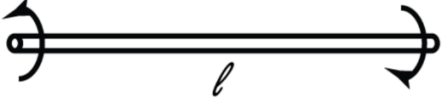
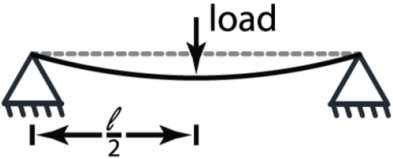
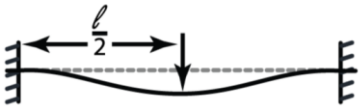
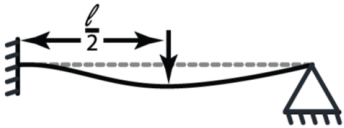
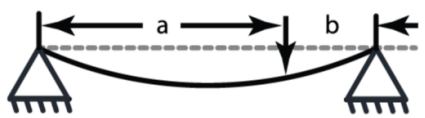
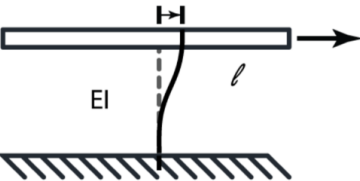
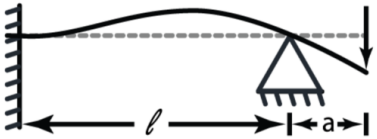
4	Mass Unbalance Induced Casing Response Ratio  $\frac{MX}{me} = \frac{\left(\frac{f}{f_n}\right)^2}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$ M = total mass, lbf s²/in m = rotor mass, lbf s²/in X = casing motion, inches peak e = rotor eccentricity, inches f = forcing frequency, Hz f_n = natural frequency, Hz ω_n = natural frequency, rad/s ζ = damping ratio = C/C_c k = spring stiffness, lbf/in C = damping constant, lbf s/in C_c = 2Mω_n = critical damping, lbf s/in	Mass Unbalance Induced Casing Response Ratio  $\frac{MX}{me} = \frac{\left(\frac{f}{f_n}\right)^2}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2\zeta \frac{f}{f_n}\right]^2}}$ M = total mass, kg m = rotor mass, kg X = casing motion, mm peak e = rotor eccentricity, mm f = forcing frequency, Hz f_n = natural frequency, Hz ω_n = natural frequency, rad/s ζ = damping ratio = C/C_c k = spring stiffness, N/m C = damping constant, N s/m C_c = 2Mω_n = critical damping, N s/m
G Fluid Film Bearings		
4	Stability Threshold $\omega_s = \omega_o \sqrt{\frac{c}{g_n}}$ ω_s = dimensionless stability threshold ω_o = journal operating speed, rad/s c = bearing radial clearance, in g_n = standard gravity, 386.1 in/s²	Stability Threshold $\omega_s = \omega_o \sqrt{\frac{c}{g_n}}$ ω_s = dimensionless stability threshold ω_o = journal operating speed, rad/s c = bearing radial clearance, m g_n = standard gravity, 9.807 m/s²

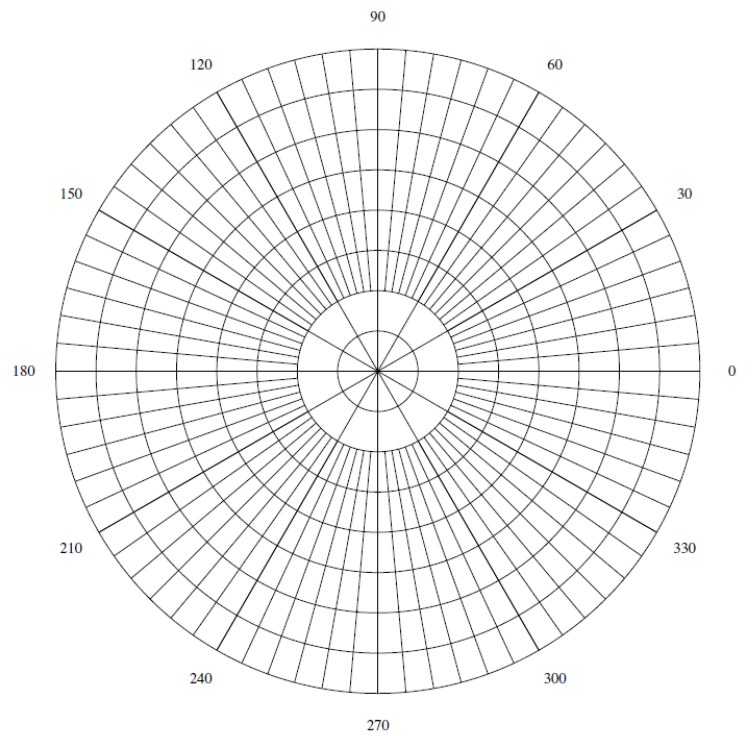
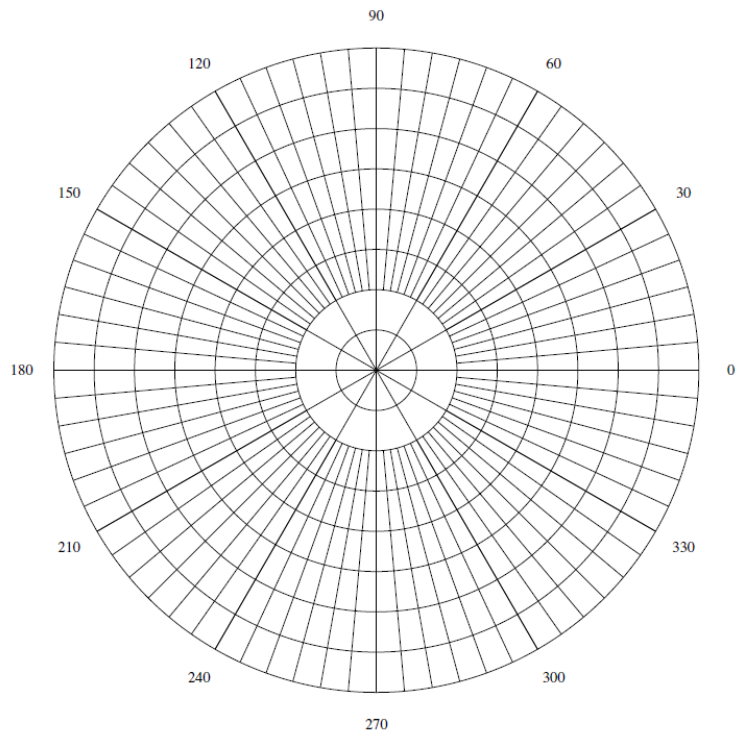
④	Sommerfeld Number $S = \frac{\mu N L D}{W} \left(\frac{R}{C} \right)^2$ S = Sommerfeld number μ = oil viscosity, lbf s/in² N = shaft speed, RPS L = bearing length, in D = journal diameter, in W = load on the bearing, lbf R = radius of the journal, in C = bearing radial clearance, in	Sommerfeld Number $S = \frac{\mu N L D}{W} \left(\frac{R}{C} \right)^2$ S = Sommerfeld number μ = oil viscosity, N s/mm² N = shaft speed, RPS L = bearing length, mm D = journal diameter, mm W = load on the bearing, N R = radius of the journal, mm C = bearing radial clearance, mm
H	AMPLIFICATION FACTOR AND DAMPING RATIO	
④	Half Power Method $AF = \frac{N_{CR}}{N_2 - N_1}$ AF = amplification factor (at resonance) N_{CR} = critical speed, RPM N₂, N₁ = half power points, RPM	Half Power Method $AF = \frac{N_{CR}}{N_2 - N_1}$ AF = amplification factor (at resonance) N_{CR} = critical speed, Hz N₂, N₁ = half power points, Hz
④	Phase Change Method $AF = \frac{\pi N_{CR}}{360} \frac{\Delta \phi}{\Delta N}$ AF = amplification factor (at resonance) N_{CR} = critical speed, RPM Δφ = phase change, degrees ΔN = speed change, RPM	Phase Change Method $AF = \frac{\pi N_{CR}}{360} \frac{\Delta \phi}{\Delta N}$ AF = amplification factor (at resonance) N_{CR} = critical speed, Hz Δφ = phase change, degrees ΔN = speed change, Hz
④	Time Domain Method $\delta = \frac{1}{n} \ln \frac{X_o}{X_n} = \text{log decrement}$ $\zeta = \frac{C}{C_c} \approx \frac{\delta}{2\pi} \approx \frac{1}{2AF} = \text{damping ratio}$ n = number of cycles $\frac{X_o}{X_n}$ = amplitude ratio in n cycles C = damping constant, lbf s/in C_c = 2Mω_n = critical damping, lbf s/in AF = amplification factor M = system mass, lbf s²/in ω_n = system natural frequency, rad/s	Time Domain Method $\delta = \frac{1}{n} \ln \frac{X_o}{X_n} = \text{log decrement}$ $\zeta = \frac{C}{C_c} \approx \frac{\delta}{2\pi} \approx \frac{1}{2AF} = \text{damping ratio}$ n = number of cycles $\frac{X_o}{X_n}$ = amplitude ratio in n cycles C = damping constant, N s/m C_c = 2Mω_n = critical damping, N s/m AF = amplification factor M = system mass, N s²/m ω_n = system natural frequency, rad/s

4	Dual Constant/Dual Channel Method Damping Calculation: $AF = \frac{\left(\frac{f_a}{f_b}\right)^2 + 1}{\left(\frac{f_a}{f_b}\right)^2 - 1}$ f_a = frequency above resonance, where the real or imaginary part of the transfer function reaches a peak f_b = frequency below resonance, where the real or imaginary part of the transfer function reaches a peak of opposite sign AF = amplification factor (at resonance) $AF \approx \frac{1}{2\frac{C}{C_c}} \approx \frac{1}{(2\zeta)}$ C = damping constant, lbf s/in C_C = 2Mω_n critical damping, lbf s/in AF = amplification factor M = system mass, lbf s²/in ω_n = system natural frequency, $\frac{\text{rad}}{\text{s}}$	Dual Constant/Dual Channel Method Damping Calculation: $AF = \frac{\left(\frac{f_a}{f_b}\right)^2 + 1}{\left(\frac{f_a}{f_b}\right)^2 - 1}$ f_a = frequency above resonance, where the real or imaginary part of the transfer function reaches a peak f_b = frequency below resonance, where the real or imaginary part of the transfer function reaches a peak of opposite sign AF = amplification factor (at resonance) $AF \approx \frac{1}{2\frac{C}{C_c}} \approx \frac{1}{(2\zeta)}$ C = damping constant, N s/m C_C = 2Mω_n critical damping, N s/m AF = amplification factor M = system mass, N s²/m ω_n = system natural frequency, $\frac{\text{rad}}{\text{s}}$																																
I	MATERIAL PROPERTIES Constants shown may be approximate, rounded, or stated for the field of vibration.																																	
4	E = Modulus of Elasticity <table><tr><td>Aluminum</td><td>10.0 x 10⁶ lbf/in²</td></tr><tr><td>Ductile Iron</td><td>24.5 x 10⁶ lbf/in²</td></tr><tr><td>Steel</td><td>29.0 x 10⁶ lbf/in²</td></tr><tr><td>Titanium</td><td>14.9 x 10⁶ lbf/in²</td></tr></table> G = Modulus of Rigidity <table><tr><td>Aluminum</td><td>3.9 x 10⁶ lbf/in²</td></tr><tr><td>Ductile Iron</td><td>9.3 x 10⁶ lbf/in²</td></tr><tr><td>Steel</td><td>11.2 x 10⁶ lbf/in²</td></tr><tr><td>Titanium</td><td>5.9 x 10⁶ lbf/in²</td></tr></table>	Aluminum	10.0 x 10 ⁶ lbf/in ²	Ductile Iron	24.5 x 10 ⁶ lbf/in ²	Steel	29.0 x 10 ⁶ lbf/in ²	Titanium	14.9 x 10 ⁶ lbf/in ²	Aluminum	3.9 x 10 ⁶ lbf/in ²	Ductile Iron	9.3 x 10 ⁶ lbf/in ²	Steel	11.2 x 10 ⁶ lbf/in ²	Titanium	5.9 x 10 ⁶ lbf/in ²	E = Modulus of Elasticity <table><tr><td>Aluminum</td><td>69 GPa</td></tr><tr><td>Ductile Iron</td><td>169 GPa</td></tr><tr><td>Steel</td><td>200 GPa</td></tr><tr><td>Titanium</td><td>103 GPa</td></tr></table> G = Modulus of Rigidity (GPa = 10⁹ Pa) <table><tr><td>Aluminum</td><td>27 GPa</td></tr><tr><td>Ductile Iron</td><td>64 GPa</td></tr><tr><td>Steel</td><td>77 GPa</td></tr><tr><td>Titanium</td><td>41 GPa</td></tr></table>	Aluminum	69 GPa	Ductile Iron	169 GPa	Steel	200 GPa	Titanium	103 GPa	Aluminum	27 GPa	Ductile Iron	64 GPa	Steel	77 GPa	Titanium	41 GPa
Aluminum	10.0 x 10 ⁶ lbf/in ²																																	
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4	Density <table><tr><td>Aluminum</td><td>0.0978 lbf/in³</td></tr><tr><td>Ductile Iron</td><td>0.2579 lbf/in³</td></tr><tr><td>Steel</td><td>0.2836 lbf/in³</td></tr><tr><td>Titanium</td><td>0.1620 lbf/in³</td></tr></table>	Aluminum	0.0978 lbf/in ³	Ductile Iron	0.2579 lbf/in ³	Steel	0.2836 lbf/in ³	Titanium	0.1620 lbf/in ³	Density <table><tr><td>Aluminum</td><td>2712 kg/m³</td></tr><tr><td>Ductile Iron</td><td>7300 kg/m³</td></tr><tr><td>Steel</td><td>7850 kg/m³</td></tr><tr><td>Titanium</td><td>4500 kg/m³</td></tr></table>	Aluminum	2712 kg/m ³	Ductile Iron	7300 kg/m ³	Steel	7850 kg/m ³	Titanium	4500 kg/m ³																
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J	AREA MOMENTS OF INERTIA <i>Figure also applies to Section K</i>	
4	Bending Area Moment of Inertia $I_x = I_y = \frac{\pi D^4}{64} \quad (\text{solid shaft})$ $I_x = I_y = \frac{\pi(D^4 - d^4)}{64} \quad (\text{hollow shaft})$ $I_x = I_y$ = bending area moment of inertia of circular cross-section, in⁴ D = outer diameter, in d = inner diameter, in	Bending Area Moment of Inertia $I_x = I_y = \frac{\pi D^4}{64} \quad (\text{solid shaft})$ $I_x = I_y = \frac{\pi(D^4 - d^4)}{64} \quad (\text{hollow shaft})$ $I_x = I_y$ = bending area moment of inertia of circular cross-section, mm⁴ D = outer diameter, mm d = inner diameter, mm
4	Polar Area Moment of Inertia $J = \frac{\pi D^4}{32} \quad (\text{solid shaft})$ $J = \frac{\pi(D^4 - d^4)}{32} \quad (\text{hollow shaft})$ J = polar area moment of inertia of circular cross section, in⁴ D = outer diameter, in d = inner diameter, in	Polar Area Moment of Inertia $J = \frac{\pi D^4}{32} \quad (\text{solid shaft})$ $J = \frac{\pi(D^4 - d^4)}{32} \quad (\text{hollow shaft})$ J = polar area moment of inertia of circular cross section, mm⁴ D = outer diameter, mm d = inner diameter, mm
K	MASS MOMENTS OF INERTIA	
4	Mass Moment of Inertia, Thin Disk $I_x = I_y = \frac{MD^2}{16}$ $I_x = I_y$ = mass moment of inertia, lbf s²/in M = mass, lbf s²/in D = diameter, in	Mass Moment of Inertia, Thin Disk $I_x = I_y = \frac{MD^2}{16}$ $I_x = I_y$ = mass moment of inertia, kg m² M = mass, kg D = diameter, m

④	Polar Mass Moment of Inertia, Shaft $J = \frac{MD^2}{8}$ J = polar mass moment of inertia, lbf s² in M = mass, lbf s²/in D = diameter, in	Polar Mass Moment of Inertia, Shaft $J = \frac{MD^2}{8}$ J = polar mass moment of inertia, kg m² M = mass, kg D = diameter, m
L	CASES OF STIFFNESS CALCULATION All cases shown apply to both unit systems	
④	Common Parameters for US Customary Usage of Case formulas k = stiffness, lbf/in k _t = torsional stiffness, lbf in/rad I = bending area moment of inertia of circular cross-section, in ⁴ J = polar area moment of inertia of circular cross section, in ⁴ D = outer diameter, in d = inner diameter, in d = rod diameter of coil, in A = cross sectional area, in ² n = number of turns, count l = total length, in E = Modulus of Elasticity, lbf/in ² G = Modulus of Rigidity, lbf/in ²	Common Parameters for SI Usage of Case formulas k = stiffness, N/m k _t = torsional stiffness, N m/rad I = bending area moment of inertia of circular cross-section, m ⁴ J = polar area moment of inertia of circular cross section, m ⁴ D = outer diameter, m d = inner diameter, m d = rod diameter of coil, m A = cross sectional area, m ² n = number of turns, count l = total length, m E = Modulus of Elasticity, Pa G = Modulus of Rigidity, Pa
④	Case A: Springs in series  $k = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}$	Case B: Springs In parallel  $k = k_1 + k_2$
④	Case C:  $k_t = \frac{EI}{\ell}$	Case D:  $k = \frac{EA}{\ell}$

4	Case E:  $k_t = \frac{GJ}{\ell}$	Case F:  $k = \frac{Gd^4}{64nR^3}$
4	Case G:  $k = \frac{3EI}{\ell^3}$	Case H:  $k = \frac{48EI}{\ell^3}$
4	Case I:  $k = \frac{192EI}{\ell^3}$	Case J:  $k = \frac{768EI}{7\ell^3}$
4	Case K:  $k = \frac{3EI\ell}{a^2b^2}$ $\ell = a + b$	Case L:  $k = \frac{12EI}{\ell^3}$
4	Case M:  $k = \frac{3EI}{(\ell+a)a^2}$	Case N:  $k = \frac{12EI}{a^2(3\ell+4a)}$



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